A seamless hybrid RANS-LES model based on transport equations for the subgrid stresses and elliptic blending

Atabak FADAI-GHOTBI
Christophe FRIESS
Rémi MANCEAU
Jacques BORÉE

Laboratoire d'Etudes Aérodynamiques
UMR 6609 CNRS/Université de Poitiers/ENSMA
Poitiers, France

Code Saturne User's Club December 2008

Introduction

- Unsteady information crucial in industry: fluid/structure interaction, thermal fatigue, noise $\Longrightarrow$ prediction of characteristic frequencies and energy contained in the dominant structures
- 3 major axis of simulation:
  - DNS: resolve all scales $\Longrightarrow$ 3D unsteady accurate solution, but unaffordable for industrial problems
  - LES: resolve large scales and model small scales. But QDNS mode in the near wall zone $\Longrightarrow$ high CPU cost
  - RANS: model all scales $\Longrightarrow$ low CPU cost but steady solution
- Lots of unsteady low cost approaches, between RANS and LES:

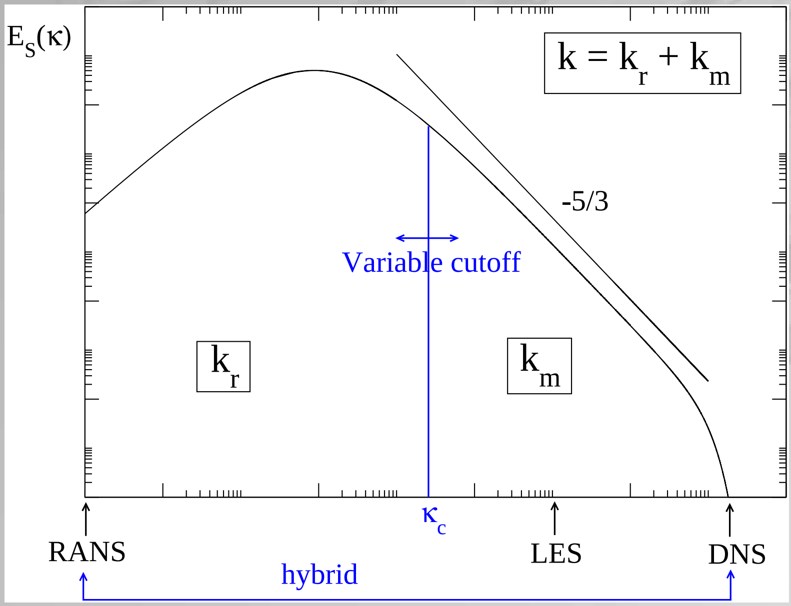
  $\Longrightarrow$ hybrid RANS-LES: VLES (Speziale 1998), LNS (Batten et al. 2002), DES (Spalart 2000), PITM (Schiestel & Dejoan 2005), ...

  $\Longrightarrow$ other approaches: SDM (Kourta & Ha Minh 1993), SAS (Menter & Egorov 2005), URANS (Iaccarino et al. 2003), ...

Introduction on hybrid RANS-LES models

Two types of hybrid approaches :

- imposed frontier: easier to model but complex coupling between RANS and LES zones
- seamless (continuous transition): simpler in practical applications, but modelling problems



- Spectral theory of turbulence: formal framework consistent for hybrid seamless models
- Compatible with the two extreme limits RANS and DNS: transition parameter ?
- Location of the cutoff in the productive zone $\Longrightarrow$ production and redistribution
- Decrease the CPU cost (coarser mesh)

Main steps

- Provide a theoretical framework to the separation resolved/modelled scales
  $\Longrightarrow$ PITM approach (Schiestel & Dejoan 2005, Chaouat & Schiestel 2005)
- Based on transport equations for the subgrid stresses
  $\Longrightarrow$ production and redistribution when the cutoff is in the energetic part of the spectrum
- Use of the near-wall RANS Elliptic Blending Reynolds Stress Model (Manceau & Hanjalić 2002, Manceau 2005)

PITM model

(Schiestel & Dejoan 2005, Chaouat & Schiestel 2005)

- Decomposition: $U_i^* = \underbrace{\tilde{U}_i(\mathbf{x},t)}_{\text{filtered velocity (resolved)}} + \underbrace{u_i''(\mathbf{x},t)}_{\text{residual fluctuation}}$
- Filtered velocity obtained by convolution product: $\tilde{U}_i = \langle U_i^* \rangle = F_{\Delta_S} * U_i^*$
- Spectral cutoff to separate resolved scales $[0, \kappa_c]$ and modelled scales $[\kappa_c, \infty]$
- Filtered equations (Germano 1992):

$$\frac{\tilde{D}\tilde{U}_i}{\tilde{D}t} = -\frac{1}{\rho}\frac{\partial \tilde{P}}{\partial x_i} + \nu \frac{\partial^2 \tilde{U}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} \qquad (1)$$

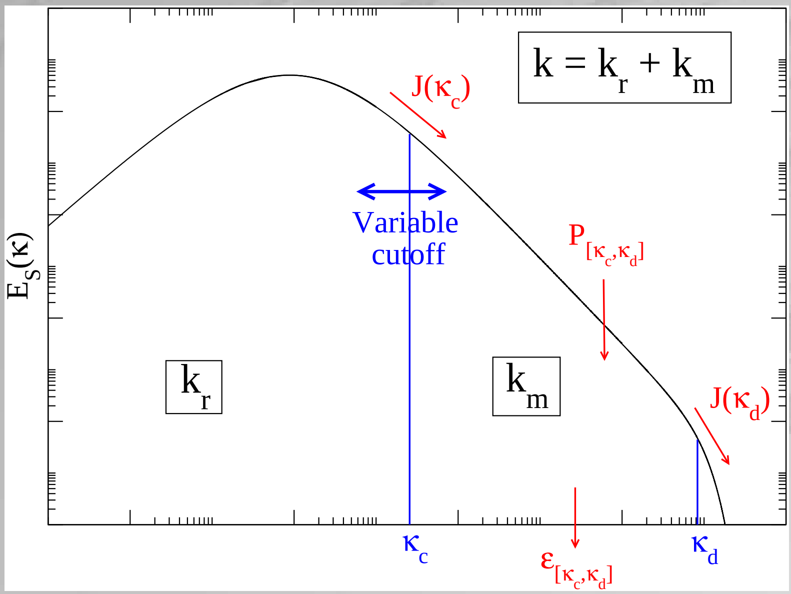
$$\frac{\tilde{D}\tau_{ij}}{\tilde{D}t} = D_{ij}^T + D_{ij}^\nu + \phi_{ij} + P_{ij} - \varepsilon_{ij} \qquad (2)$$

$\tau_{ij} = \left\langle U_i^* U_j^* \right\rangle - \langle U_i^* \rangle \left\langle U_j^* \right\rangle$: characterizes the influence of the filtered (small) scales on the resolved (large) scales

- Aim: retrieve the classical form of the RANS equation for ε + modify the coefficients to characterize only the modelled scales

- 3 zones: $[0, \kappa_c]$, $[\kappa_c, \kappa_d]$, $[\kappa_d, \infty]$. Integration of the energy spectrum equation on $[\kappa_c, \kappa_d]$ gives the evolution of $k_m = \frac{1}{2}\overline{\tau_{ii}}$:

$$\frac{\partial k_m}{\partial t} = \underbrace{P_{[\kappa_c,\kappa_d]} + \mathcal{J}(\kappa_c)}_{P_m} - \underbrace{(\varepsilon_{[\kappa_c,\kappa_d]} + \mathcal{J}(\kappa_d))}_{\varepsilon_m} \tag{3}$$



- Assumption (Schiestel 1983):

$$\kappa_d = \kappa_c + \chi \frac{\varepsilon_m}{k_m^{3/2}} \tag{4}$$

- Time derivative of (4) gives:

$$\frac{\partial \varepsilon_m}{\partial t} = C_{\varepsilon_1} \frac{P_m \varepsilon_m}{k_m} - C^*_{\varepsilon_2} \frac{{\varepsilon_m}^2}{k_m} \tag{5}$$

$$C^*_{\varepsilon_2} = C_{\varepsilon_1} + \frac{k_m}{k}(C_{\varepsilon_2} - C_{\varepsilon_1})$$

	DNS	RANS
$f_k = k_m/k$	0	1

Subgrid scale model : EB-RSM

(Manceau & Hanjalić 2002, Manceau 2005)

- Inspired by Durbin's elliptic relaxation theory (1991), taking into account the inviscid and non-local blocking effect of the wall
- Simpler (only 1 more elliptic equation to resolve, instead of 6)

$$\boxed{\alpha - L_{SGS}^2 \nabla^2 \alpha = 1} \tag{6}$$

EB-RSM model blends the near-wall and far from the wall variables (ε_{ij} & ϕ_{ij}) as:

$$X_{ij} \quad = \quad (1-\alpha^2) X_{ij}^w + \alpha^2 X_{ij}^h \tag{7}$$

- No more explicit dependency on distance to the wall $\Longrightarrow$ useful in complex geometries
- Valid in unsteady approaches (same asymptotic behaviour + non-local effect)
- Easy to implement in an existing code + robust

Choice of the length scale of wall effects

Blocking effect: consequence of the incompressibility of the fluctuating field

$\Longrightarrow$ in the hybrid context, blocking effect must be imposed only on the modelled scales $\Longrightarrow L_p \searrow$

$$\boxed{\alpha - L_{SGS}^2 \nabla^2 \alpha = 1} \tag{8}$$

- EB-RSM in the RANS framework:

$$L_p = C_L \max\left(\frac{k^{3/2}}{\varepsilon}, C_\eta \frac{\nu^{3/4}}{\varepsilon^{1/4}}\right) \tag{9}$$

- EB-RSM in the hybrid framework:

$$L_{SGS} = C_L \max\left(\frac{k_{SGS}^{3/2}}{\varepsilon}, C_\eta f_k^{3/2} \frac{\nu^{3/4}}{\varepsilon^{1/4}}\right) \tag{10}$$

$\Longrightarrow$ consistent with both RANS ($L_{SGS} \to L_p$) and DNS ($L_{SGS} \to 0$) limits

Choice of $f_k = k_{SGS}/k$

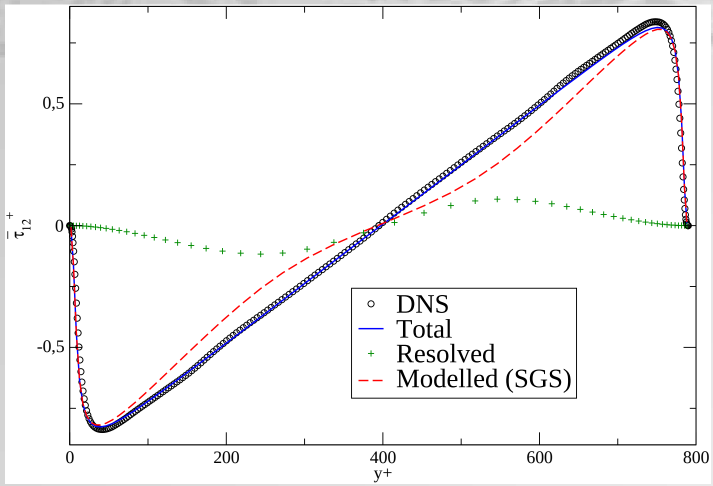
Using Kolmogorov's law for the spectrum: $\boxed{f_k(\mathbf{x},t) = (C\kappa_c L)^{-2/3}}$

Not consistent with the RANS limit ($f_k = 1$) at the wall $\Longrightarrow$ need for a blended formulation

$$f_k = (1-\alpha^2) + \alpha^2 (C\kappa_c L)^{-2/3}$$

Transition RANS / LES is controlled by

- mesh size ($\kappa_c = \pi/\Delta$)
- distance to the wall (implicitely contained in α)



Numerical aspects

Calibration in channel flow at $Re_\tau = 395$. Box size 4H*H*2H , cartesian mesh

N_{cell}	Δx^+	Δz^+	Δy_1^+	Δy_c^+
55 296	100	50	3	40

Coarse mesh, in the sense of classical LES.

Discretization and schemes :

- Time : $\Delta t^+ = 3.5\ 10^{-6}$; Crank-Nicholson (2nd order)
- Space :
  - centered (2nd order) for velocities
  - upwind (1st order) for subgrid stresses

PITM and unsteadiness issues

Macro-fluctuations of strain-rate $\Longrightarrow$ PITM model unable to respond
$\Longrightarrow$ any calculation degenerates to a steady solution

Ideas :

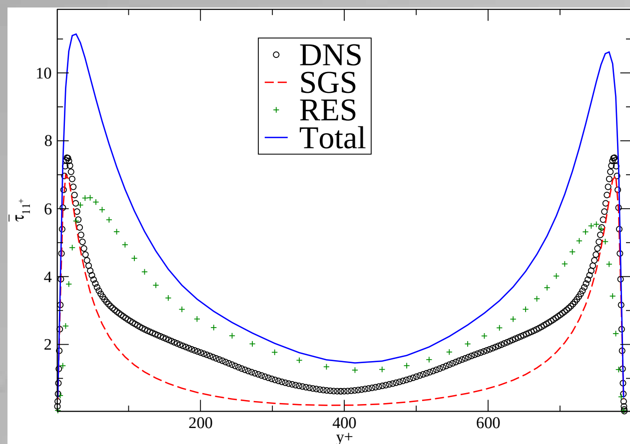
- Smooth strain fluctuations, e.g. by averaging source terms in homogeneous directions (x,z)
  - strain-rate
  - production
  - rotation-rate

  $\rightarrow$ First approach, easy to implement, but physically unjustified (averaging of production...)

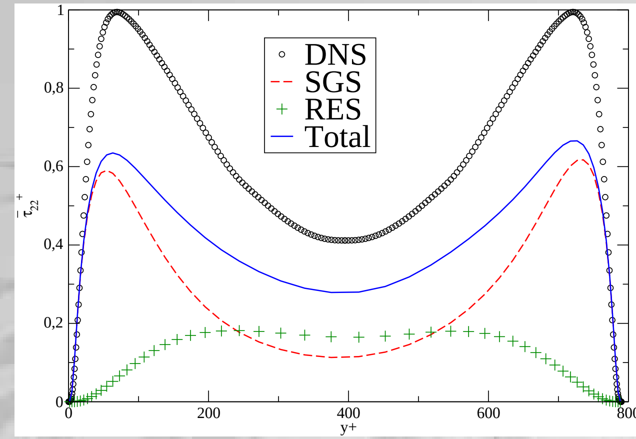
- Dynamical approach: to better pilot k_{SGS} and thus f_k
  - change $C^*_{\varepsilon 2}$: to avoid increasing of k_{SGS} , $\varepsilon \nearrow$, e.g. by $C^*_{\varepsilon 2} \searrow$

  $\rightarrow$ Physically better, but sophisticated and still empirical

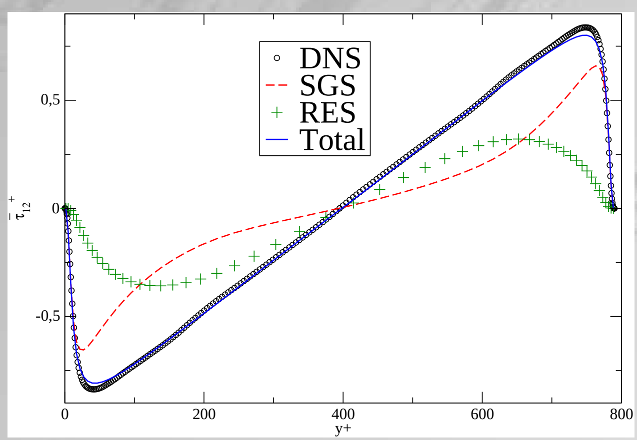
Results with source terms averaging approach (1/2)



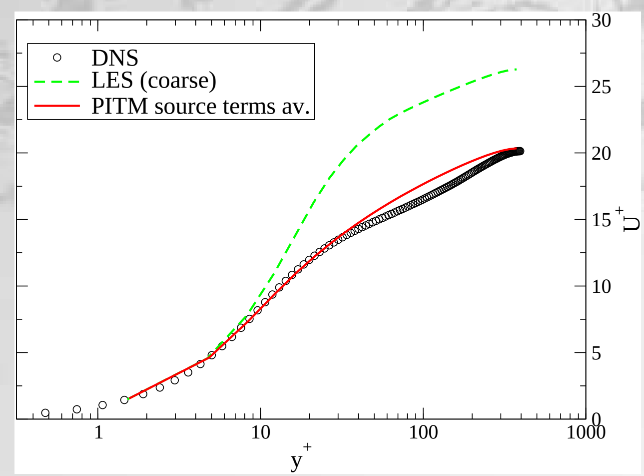
τ_{11}^{+}



τ_{22}^{+}

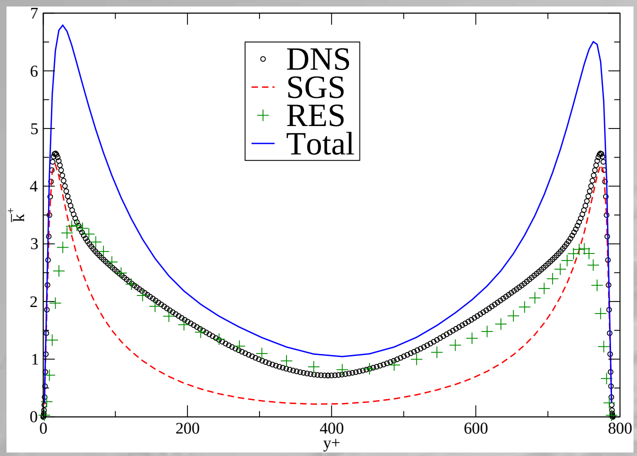


τ_{12}^{+}

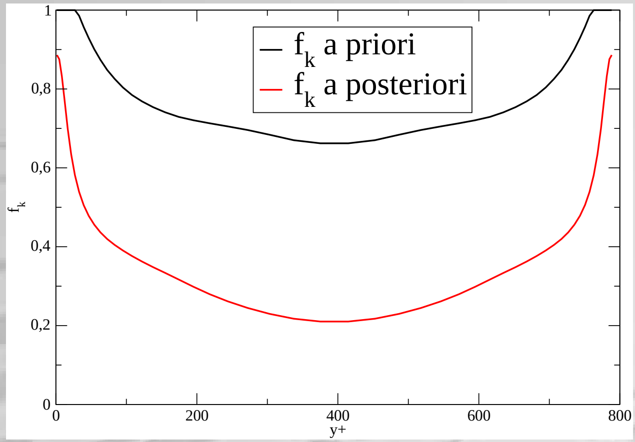


U^{+}

Results with source terms averaging approach (2/2)



k^+



f_k

- Erroneous anisotropy (τ_{11} overestimated, τ_{22} underestimated)
- f_k observed *a posteriori* is lower than the f_k applied *a priori* $\Longrightarrow$ peak of resolved energy in the buffer layer, showing the necessity to better control f_k

Dynamical approach

AIM : Keep the **observed** rate of modelled turbulent kinetic energy f_k^o close to its **target**, f_k^t

$$f_k^o = \frac{\overline{k_{SGS}}}{\overline{k_{SGS}} + \overline{k_{LES}}} = \frac{\overline{k_{SGS}}}{\overline{k_{TOT}}} \quad (11)$$

$$f_k^t = (1-\alpha^2) + \alpha^2 (C\kappa_c L)^{-2/3} \quad (12)$$

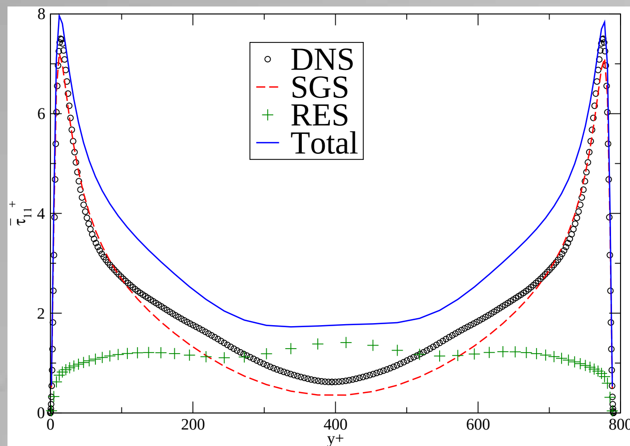
Idea : Control k_{SGS} by modifying $C_{\varepsilon 2}^* \Longrightarrow \delta C_{\varepsilon 2}^*$ function of the gap between f_k^o and f_k^t

The analysis suggests:

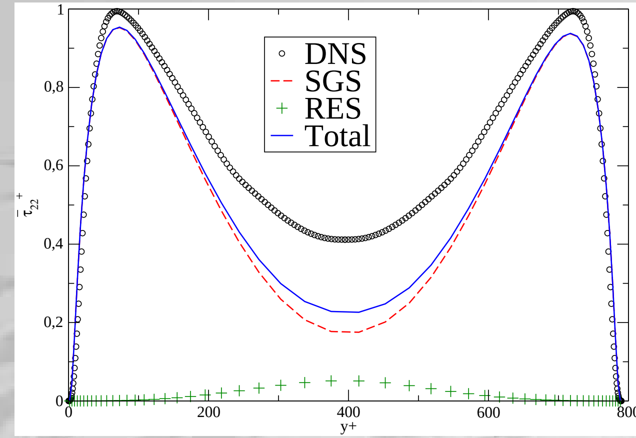
$$\delta C_{\varepsilon 2}^* = \aleph(\frac{f_k^t}{f_k^o} - 1), \aleph > 0 \quad (13)$$

$f_k^o = f_k^t \Longrightarrow \delta C_{\varepsilon 2}^* = 0$; $f_k^o > f_k^t \Longrightarrow \delta C_{\varepsilon 2}^* < 0$; $f_k^o < f_k^t \Longrightarrow \delta C_{\varepsilon 2}^* > 0$

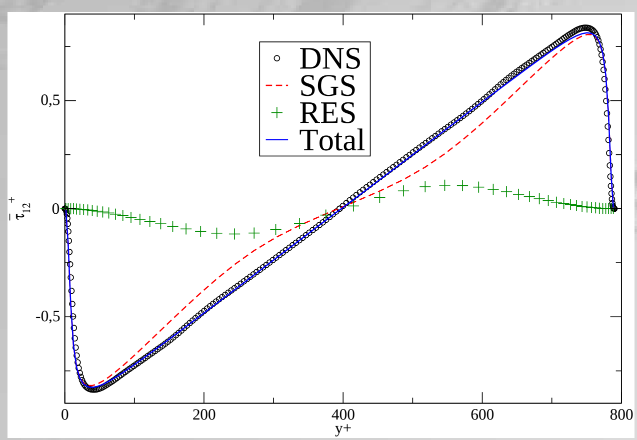
Results with the dynamical approach (1/2)



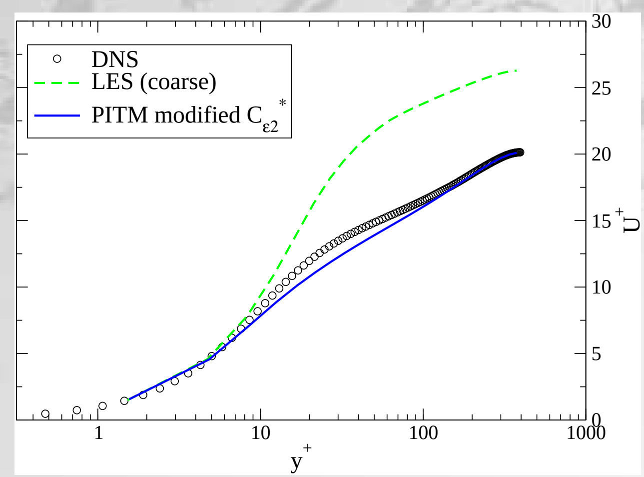
τ_{11}^{+}



τ_{22}^{+}

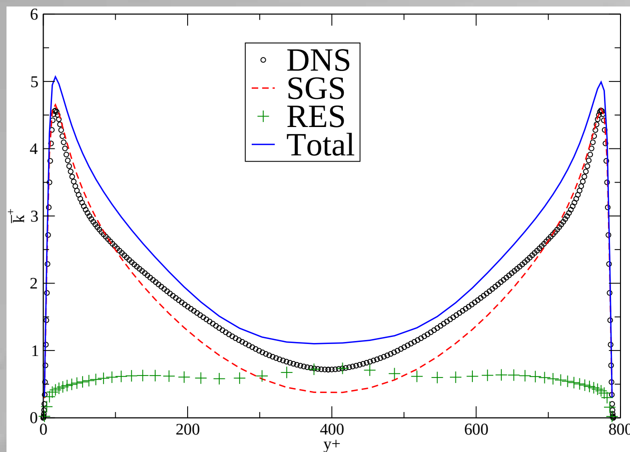


τ_{12}^{+}

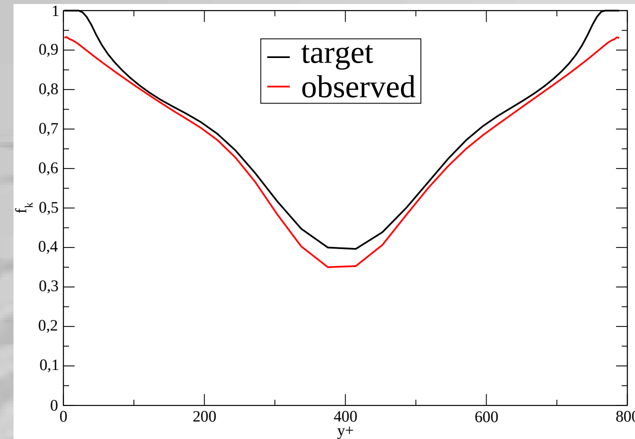


U^{+}

Results with the dynamical approach (2/2)



k^+



f_k

- Velocity profile improved, but slightly underestimated in the log-zone
- Peaks of turbulent stresses better predicted, but still wrong anisotropy towards the center of the channel
- $f_k^o \approx f_k^t$

Conclusions

- The spectral partitionning gives a consistent formal framework to bridge RANS, LES and DNS in a continuous manner
- Use of transport equations for τ_{ij} to take into account production and redistribution when the cutoff is in the productive zone of the spectrum $\Longrightarrow$ LES on coarse mesh
- Encouraging results:
  - Transition well represented ; SGS part dominant near the wall
  - With the modified $C^*_{\varepsilon 2}$ procedure, peaks of total turbulent stresses are well predicted, showing the validity of the EB-RSM in a hybrid context
- Need validation on finer meshes (comparison with LES, on typical meshes)
- Further work : higher Reynolds number + rotating channel + separated flows