Wall treatment in Large Eddy Simulation

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Overview

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2. Quick review of existing approaches
3. Extending the classical wall-model
4. Results on a heated channel flow
5. Conclusion and perspectives

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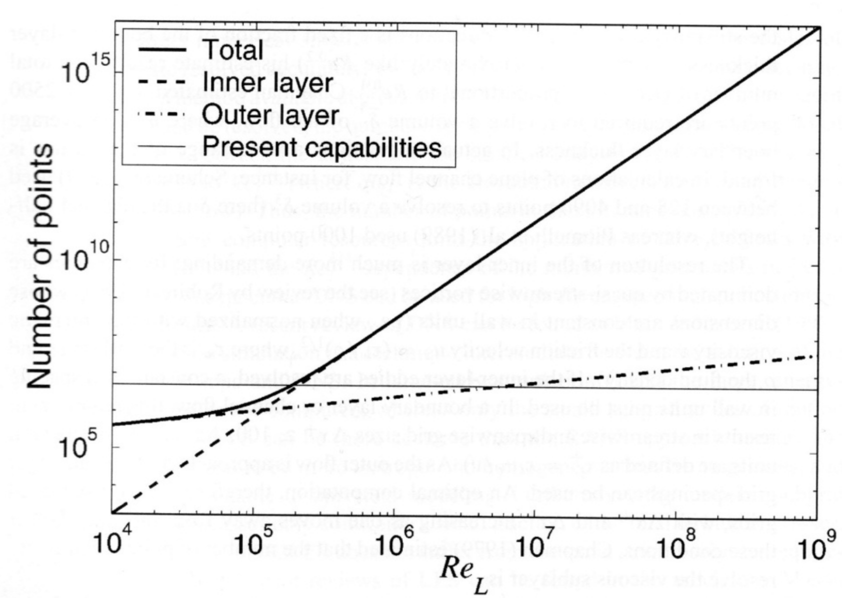
Context : Turbulence and Large-Eddy Simulation

Large Eddy Simulation

- Ideal to solve unsteady flows
  - Useful for areas like Fluid Structure Interaction, thermal fatigue
- Resolve large scales while modelling smallest ones
- But, need for a very fine mesh for turbulent flows to resolve small scales next to the walls
- Spalart (2000): computers not efficient enough before 2050

Why a model for the wall boundary layer

- Estimation of the number of points needed for a computation in the different layers
  - Outer layer, nb of nodes ~ $Re^{0.5}$
  - Inner layer, nb of nodes ~ $Re^{2.4}$



Code_Saturne and its wall treatment

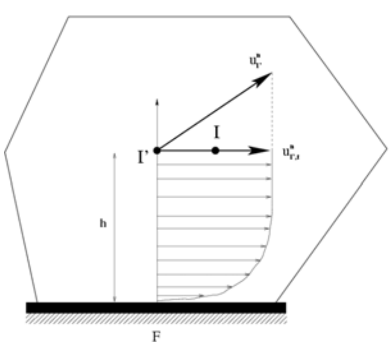
Code developped in EDF and originally designed for nuclear vessels computation

- Finite volume method on polyhedron unstructured meshes
- Collocated cell-centered variables
- Incompressible or weakly compressible Navier-Stokes equations

Two kind of wall-boundary conditions for the velocity

- Local coordinates defined by cell-centered velocity → $u_I^n \geq 0$
- Computation of the diffusive terms at the wall
  - Shear stress $|\tau_w| = \rho (u^*)^2$
- Computation of eddy viscosity
  - Cell-centered velocity gradient

$$\left.\frac{\partial u}{\partial y}\right|_{I,numerical} = \left.\frac{\partial u}{\partial y}\right|_{I,theoretial}$$



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From Schumann's law to zonal models : a quick review of classical methods

Differents available approaches

Classical « instantaneous » methods

Taking into account the « driving » terms (pressure gradient, time derivative,…)

Some classical wall-functions

Instantaneous logarithmic law

$$u^+ = \frac{u_I^n}{u_\tau} = \frac{1}{\kappa}\ln\left(\frac{yu_\tau}{\nu}\right) + B, y^+ \geq y^+_{\lim}$$

Werner & Wengle power law (shear stress directly available)

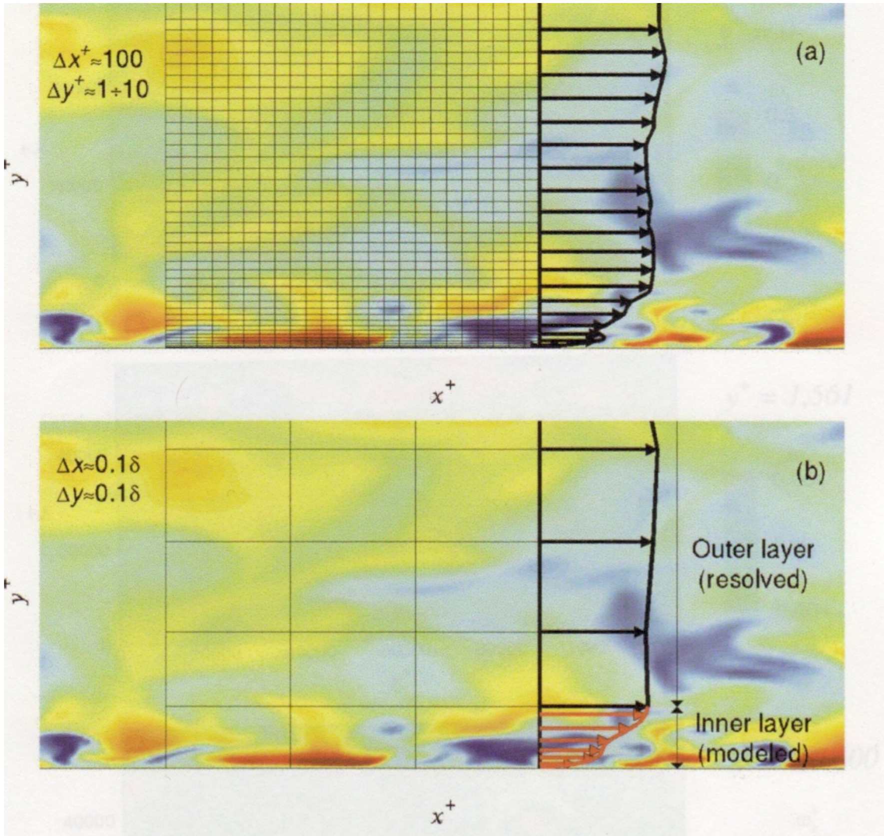
$$u^+ = \frac{u_I^n}{u_\tau} = A\left(\frac{yu_\tau}{\nu}\right)^B, y^+ \geq y^+_{\lim}$$

Reichardt law (blended law)

$$u^+ = \frac{u_I^n}{u_\tau} = \frac{1}{\kappa}\ln\left(1+\kappa y^+\right) + C\left[1-\exp\left(-\frac{y^+}{D}\right) - \frac{y^+}{D}\exp\left(-by^+\right)\right], \forall y^+$$

Thin Boundary Layer Approach (TBLE)

- Framework
  - Keep a coarse mesh for the LES
  - Solve a simplified set of equations on a 1D-mesh in the first cell next to the wall
  - Balaras *et al*, Wang, Moin
- Drawbacks
  - Rather difficult to implement on unstructured meshes



From Piomelli *et al*

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An extension of the classical wall model

Dimensionless boundary layer equations

All the terms are made dimensionless by the friction velocity.

$$u_\tau = \sqrt{\frac{|\tau_w|}{\rho}}$$

We use classical hypothesis by neglecting the following terms (in a first attempt):

- Diffusion along streamwise and spanwise directions
- Convection
- Time derivative

This leads to the following equation where F stands for the pressure gradient, time derivative,…

$$\frac{\partial}{\partial y^+}\left[\left(1+\nu_t^+\right)\frac{\partial u^+}{\partial y^+}\right] = F^+$$

Which model for the eddy viscosity ?

Mixing-lenght hypothesis

$$\nu_t^+ = L^+ U^+$$ **with** $$L^+ = \kappa y^+ \left(1 - \exp\left(-\frac{y^+}{A^+}\right)\right)$$ **and** $$U^+ = L^+ \left|\frac{\partial u^+}{\partial y^+}\right|$$

$$\nu_t^+ = \left(L^+\right)^2 \left|\frac{\partial u^+}{\partial y^+}\right|$$

Thus we can now obtain an equation on the velocity in the first cell off-wall.

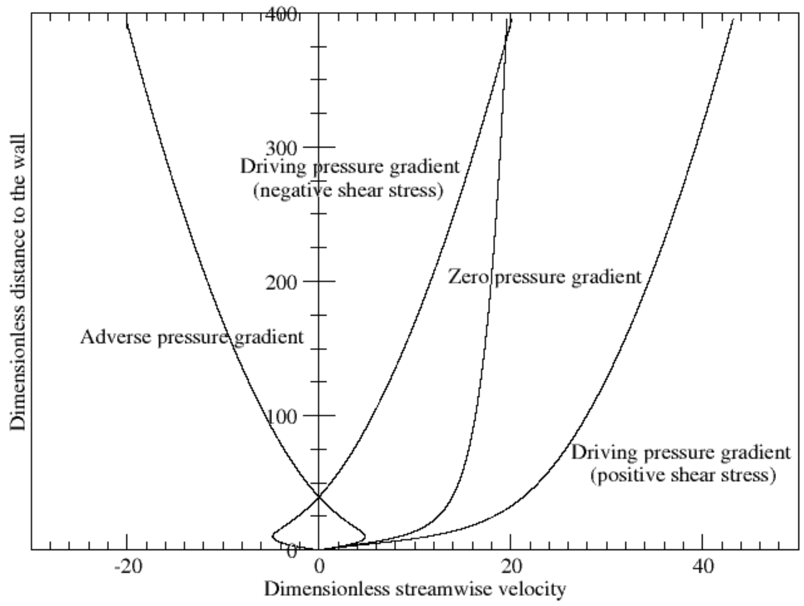
$$\frac{\partial}{\partial y^+}\left[\left(1+\nu_t^+\right)\frac{\partial u^+}{\partial y^+}\right] = F^+ \longrightarrow \left(L^+\right)^2 \left|\frac{\partial u^+}{\partial y^+}\right| \frac{\partial u^+}{\partial y^+} + \frac{\partial u^+}{\partial y^+} = F^+ y^+ + \tau_w^+$$

Velocity profiles

The study of the equation (on the right) gives the following profiles for the velocity in the first cell off-wall.

$$\left(L^+\right)^2 |X| X + X - \left(F^+ y^+ + \tau_w^+\right) = 0$$

$$u^+\left(y^+\right) = \int_0^{y^+} f\left(\xi, F^+, \tau_w^+\right) d\xi$$



Don't forget that :

- The velocity is positive in local coordinates
- The reduced shear stress is +/-1

Solutions of the 2nd order equation

1. First, $|X| = X$ → $\left(L^+\right)^2 X^2 + X - \left(F^+ y^+ + 1\right) = 0$

$$X = \frac{2\left(F^+ y^+ + 1\right)}{1 + \sqrt{1 + 4L^2\left(y^+\right)\left(F^+ y^+ + 1\right)}}$$

$$\boxed{u^+ = \int_a^b \frac{2\left(F^+ \xi + 1\right)}{1 + \sqrt{1 + 4L^{+2}\left(\xi\right)\left(F^+ \xi + 1\right)}} d\xi + u(a)}$$

2. Second, $|X| = -X$ → $-\left(L^+\right)^2 X^2 + X - \left(F^+ y^+ + 1\right) = 0$

$$X = \frac{2\left(F^+ y^+ + 1\right)}{1 + \sqrt{1 - 4L^2\left(y^+\right)\left(F^+ y^+ + 1\right)}}$$

$$\boxed{u^+ = \int_a^b \frac{2\left(F^+ \xi + 1\right)}{1 + \sqrt{1 - 4L^{+2}\left(\xi\right)\left(F^+ \xi + 1\right)}} d\xi + u(a)}$$

Velocity profiles, following the datas

	$F < 0$	$F \geq 0$
$\tau_w < 0$	Impossible, due to the constraint that the velocity must be positive at the first-cell off wall	$u^+ = \int_0^{y^*} \frac{2(F^+\xi - 1)}{1+\sqrt{1-4L^{+2}(\xi)(F^+\xi-1)}} d\xi$ $u^+ = u^+(y^*) + \int_{y^*}^{y^+} \frac{2(F^+\xi - 1)}{1+\sqrt{1+4L^{+2}(\xi)(F^+\xi-1)}} d\xi$
$\tau_w > 0$	$u^+ = \int_0^{y^*} \frac{2(F^+\xi + 1)}{1+\sqrt{1-4L^{+2}(\xi)(F^+\xi+1)}} d\xi$ $u^+ = u^+(y^*) + \int_{y^*}^{y^+} \frac{2(F^+\xi + 1)}{1+\sqrt{1+4L^{+2}(\xi)(F^+\xi+1)}} d\xi$	$u^+ = \int_0^{y^+} \frac{2(F^+\xi + 1)}{1+\sqrt{1+4L^{+2}(\xi)(F^+\xi+1)}} d\xi$

Getting the velocity friction

After having obtained the dimensionless velocity profiles, one compute the velocity friction by a Newton-Raphson method.

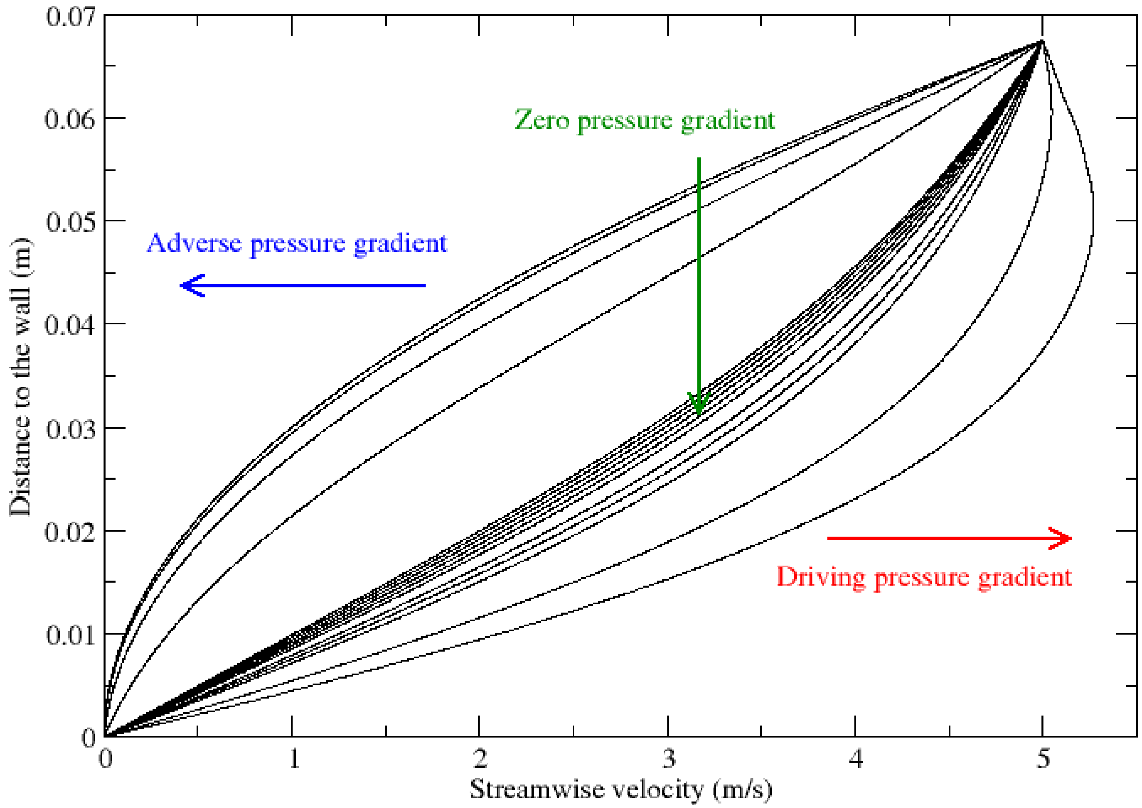
From the velocity profile $u^+(y^+) = f(y^+)$

one can obtain an equation on the friction velocity $\frac{u_I^n}{u^*} = f\left(\frac{hu^*}{\nu}\right)$

$$u^* f\left(\frac{hu^*}{\nu}\right) - u_I^n = 0 \quad \text{or} \quad h^+ f\left(h^+\right) - \mathrm{Re}_{loc} = 0$$

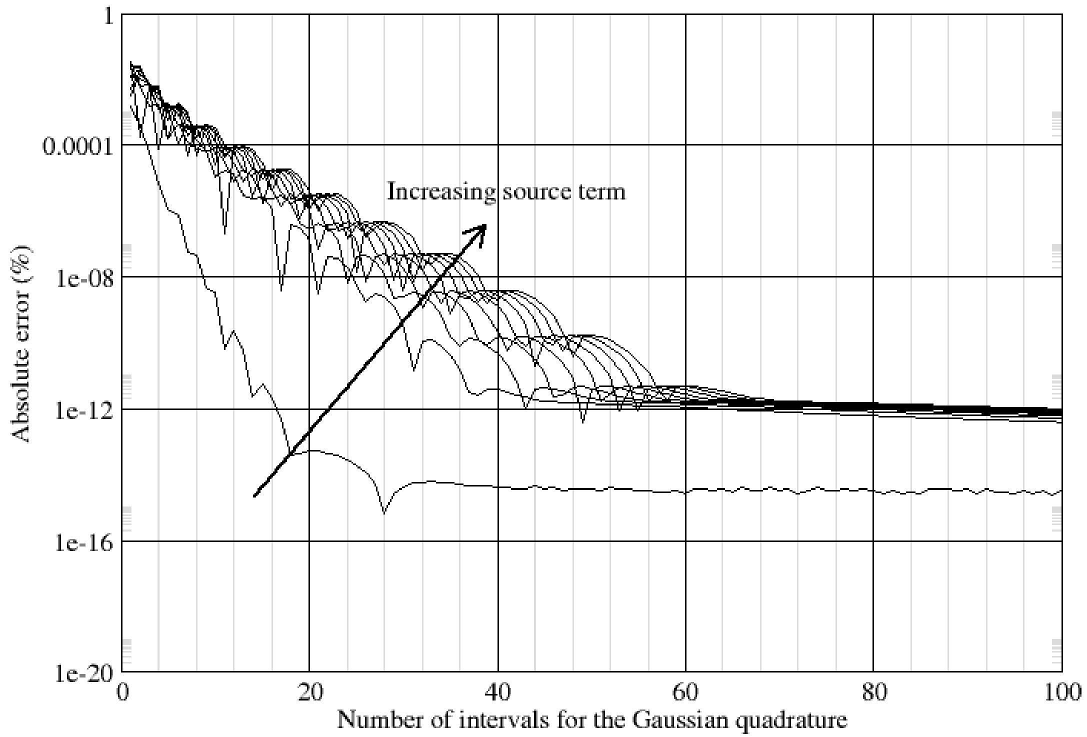
Some velocity profiles…

.. for positive shear stress and a given local Reynolds number



Numerical sensitivity

The algorithm depends only on the number of intervals and reaches an asymptote



Extension to a generic scalar case

Let's consider a convection-diffusion equation for a scalar :

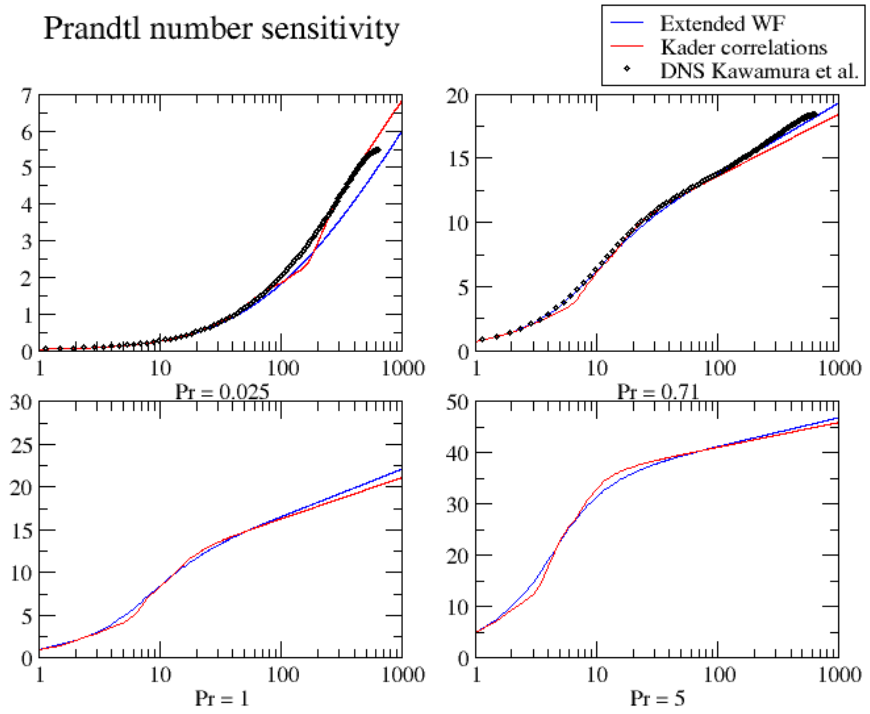
$$\frac{\partial}{\partial y}\left[(\alpha+\alpha_t)\frac{\partial \phi}{\partial y}\right] = F_\phi$$

where F stands for the unsteady, convective and source terms. The same approach leads to compute a friction scalar at the wall ϕ such as :

$$\rho u^* \phi^* = \alpha \left| \left. \frac{\partial \phi}{\partial y} \right|_{y=0} \right|$$

Prandtl number sensitivity

Rather good agreement between the predicted profiles for the extended wall-function in the generic scalar case (e.g. the temperature) and the Kader correlations.



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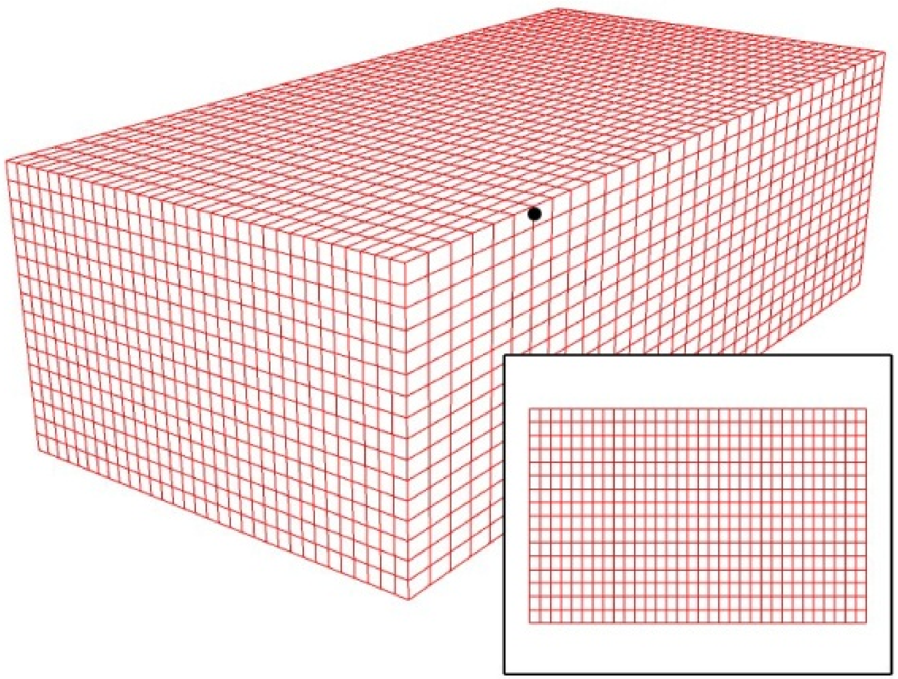
A test case :
The heated channel flow

Test case: Heated channel flow

Periodic channel: 2π x 2 x π

Mesh: 32x16x32 cells

- Uniform in all directions
- h+ = 30 at the wall
- dx+ = 80, dz+ = 40

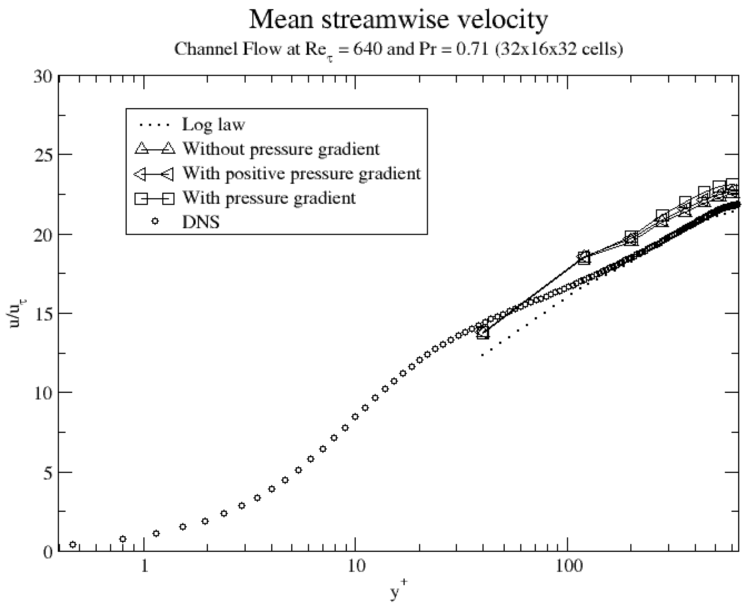


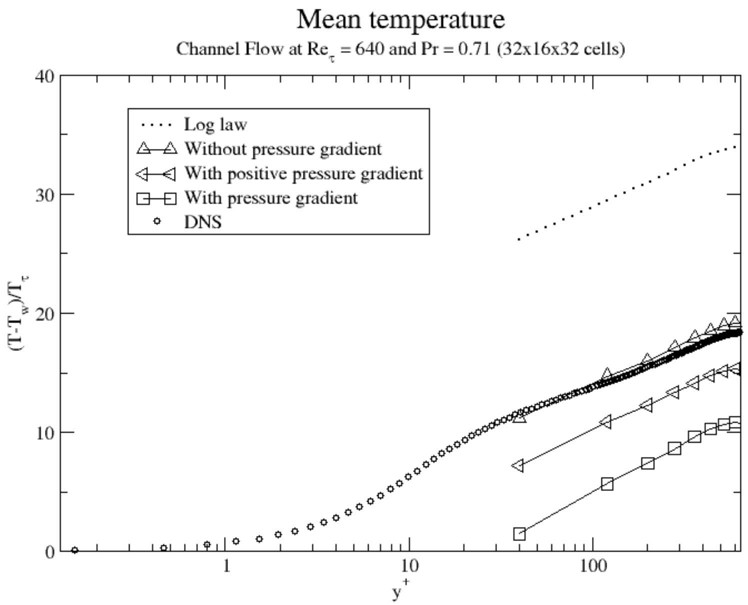
Case parameters:

- Re* = 640
- Pr = 0.71

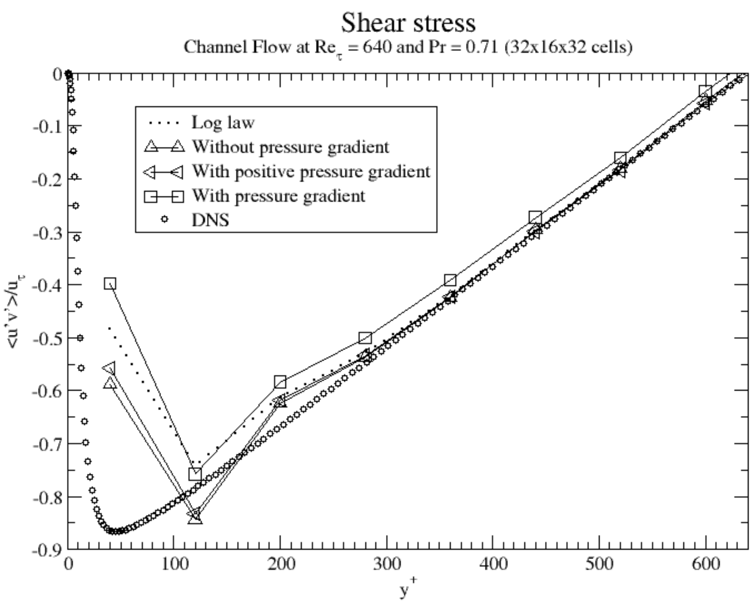
DNS from Kawamura *et al*

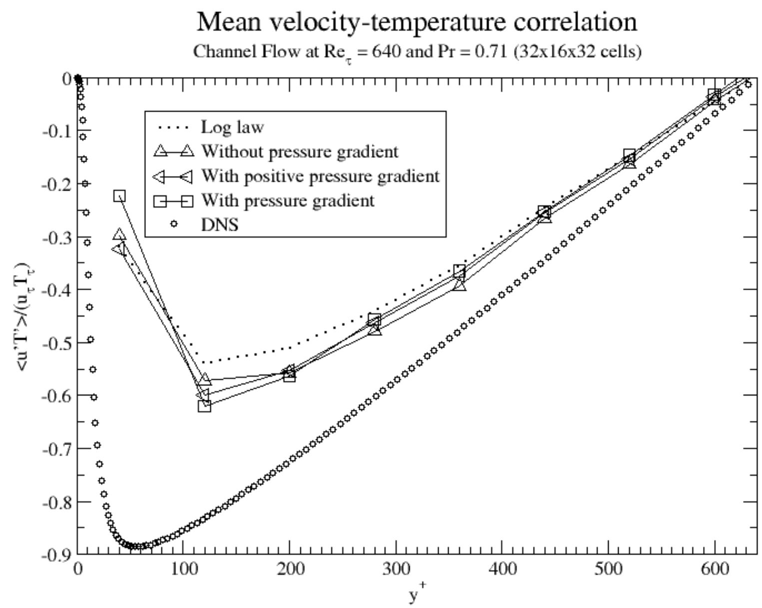
Results for a heated channel flow (1)





Results for a heated channel flow (2)

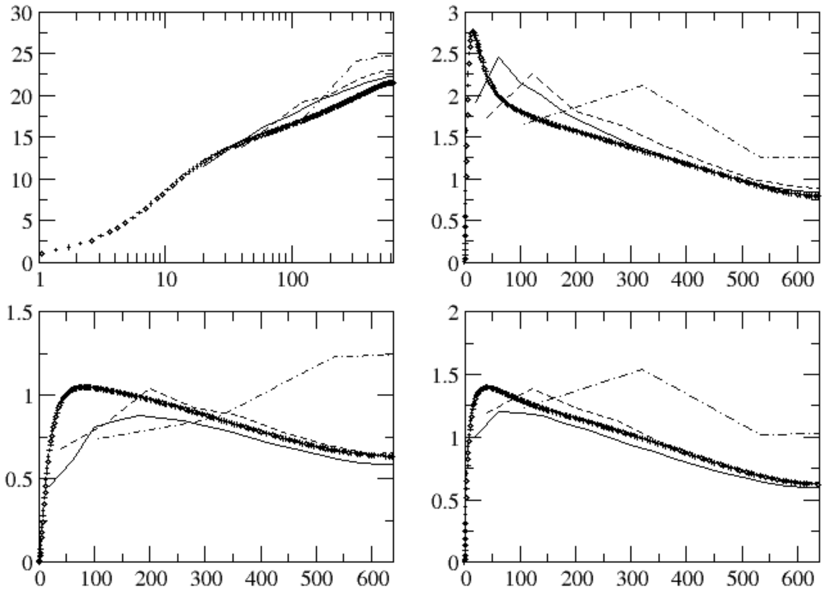




Extended law response

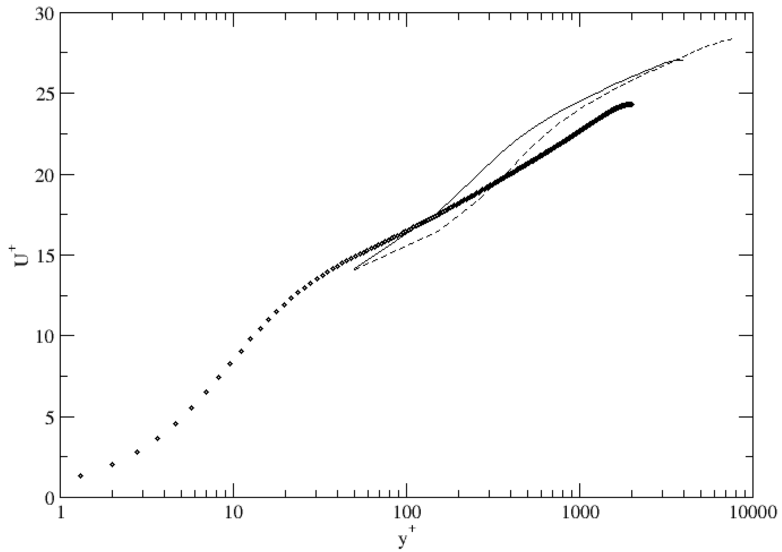
Distance to the center of the first cell off-wall:

h+ = 20, 40 and 100



Scalability against the Reynolds number:

Symbols: Re*=2000 (DNS from Jimenez)
Plain: Re*=4000
Dashed: Re*=8000



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Conclusions

Conclusions and perspectives

•Conclusions

- Quite accurate in the core of the flow and in the prediction of the shear stress near the wall
- **Meshless approach: no need to solve a system of 1D equations**
- Pressure gradient and unsteady term can be proposed by default

•Perspectives

- Apply this approach to RANS and unsteady RANS models (k-epsilon, …) ?
- Extend to buoyancy-driven flows
- Extensive testing for the scalar case