Analysis of the results

It is assumed that the standard verifications have been carried out as recommended in the previous cards (convergence in time and space, coherence of the mesh with the selected turbulence model, conservation of the quantities that shall be conserved). In the present document, the physical relevance of the results obtained with the selected modelling will be examined. It is advisable to pay attention to the following points.

A posteriori verification of the hypotheses

- Verify *a posteriori* that the hypotheses adopted when selecting the models are effectively valid: in particular, evaluate the relevant non-dimensional numbers (Reynolds, Rayleigh, Froude… y^+ for the mesh at the wall…) from the results of the calculation.

Length- and time-scales

- The integral length-scale: it can be approximately evaluated from RANS results as $L_T = \alpha\, k^{3/2}/\varepsilon$ with α ranging approximately from 0.1 to 0.3, k the turbulent kinetic energy and ε the associated dissipation. L_T represents the size of the large structures: in particular, it must be smaller than the characteristic size of the computational domain. A significantly too large value for L_T may indicate that the turbulence model is not well suited: if a first-order model has been selected (k-epsilon or k-omega), one may envisage to use a second-order model (Rij).
    - In a channel flow with a hydraulic diameter D_h, the integral length-scale is $L_T = \min(0.42\ y;\ 0.1\ D_h)$, with y standing for the distance from the wall.
    - For shear flows (jets or wakes), one may consider as a first approximation (Rodi 1984) that L_T is 10% of the width δ of the shear layer (δ being defined as the distance between the two points located on both sides of the shear layer and where the velocity differs of 1% of the velocity at infinity; for symmetrical flows, δ is the distance between the axis of symmetry and the point where the velocity differs of 1% of the velocity at infinity).
- The time-scale for turbulence: it can be evaluated approximately from RANS results as k/ε, with k the turbulent kinetic energy and ε the associated dissipation. It indicates the life-time of a turbulent structure. Using k/ε and the mean convective velocity U, one may determine the length necessary for the turbulence to develop: $U\ k/\varepsilon$.

Turbulent variables

- The ratio ν_t/ν: it is the ratio between the turbulent dynamic viscosity and the molecular dynamic viscosity, for calculations using the high Reynolds k-epsilon model. For calculations with wall-functions, this ratio should be $\nu_t/\nu > 10$ at the wall, if the first cell adjacent to the wall is large enough (except at singular points: detachment, reattachment…). This condition corresponds approximately to $y^+ > 25$ (indeed, in a plane channel flow, the following relation holds: $\nu_t = 0.42\ y\ u^*$). Otherwise, the boundary layer representation is probably wrong and it may be necessary to modify the mesh or at least to investigate the relevance of the modelling at the wall.

Correlations

- Head losses: when head losses can be evaluated *a priori* from correlations (Idel'cik 1960), check that the code provides pressure variations in reasonable agreement (a difference of 20% may be considered acceptable). A significant deviation may indicate that the mesh is not fine enough or that the turbulence modelling is not satisfying. Simplified formulae are provided hereafter to evaluate the pressure loss ΔP in some specific cases for an incompressible flow with a density ρ:
    - Smooth pipe of hydraulic diameter D_h, of length L, for an established ($L/D_h > 50$ or 100) and turbulent (Reynolds $Re = UD_h/\nu > 5000$) flow: a rough approximation of the pressure loss is $\Delta P/(\frac{1}{2} \rho U^2) = 0.02\ L/D_h$ and more accurate correlations read:

$$\Delta P/(\tfrac{1}{2} \rho U^2) = 0.3164\ Re^{-0.25}\ L/D_h \qquad \text{for } 5\,000 < Re < 30\,000$$

$\Delta P/(½\ \rho\ U^2) = 0.184\ Re^{-0.20}\ L/D_h$ for 30 000 < Re < 1 000 000

- Sudden pipe expansion from a section S_A (velocity U_A) to a section S_B:

  $\Delta P/(½\ \rho\ U_A^2) = ((S_B\text{-}S_A)/S_B)^2$

- Sudden pipe contraction from a section S_A to a section S_B (velocity U_B) :

  $\Delta P/(½\ \rho\ U_B^2) = ½\ ((S_A\text{-}S_B)/S_A)$

- Smooth pipe bend at 90°, with a curvature radius R and a diameter D_h, with a circular or a square section: very approximately, one may remember that $\Delta P/(½\ \rho\ U^2)$ decreases from 1.20 to 0.22 for R/D_h varying from 0.5 to 1.0 and remains approximately constant for R/D_h varying from 1.0 to 5.0; for a pipe bend at 180°, one should multiply the pressure loss by 1.4 approximately.
  More precisely (Idel'cik 1960, p.192), the pressure loss consists of a term due to the bend (decreasing with R/D_h) and of a term due to the friction (increasing with R/D_h):
  - For a bend at 90°:

  $\Delta P/(½\ \rho\ U^2) = 0.21/(R/D_h)^{2,5} + 0.00035\text{x}\ 90\ R/D_h$ for $0.5 < R/D_h < 1.0$

  $\Delta P/(½\ \rho\ U^2) = 0.21/(R/D_h)^{1/2} + 0.00035\text{x}\ 90\ R/D_h$ for $1.0 < R/D_h$

  - For a bend at 180°:

  $\Delta P/(½\ \rho\ U^2) = 1.4\text{x}0.21/(R/D_h)^{2.5} + 0.00035\text{x}180\ R/D_h$ for $0.5 < R/D_h < 1.0$

  $\Delta P/(½\ \rho\ U^2) = 1.4\text{x}0.21/(R/D_h)^{1/2} + 0.00035\text{x}180\ R/D_h$ for $1.0 < R/D_h$

- Heat exchange: when the heat exchange can be evaluated from simple correlations (Sacadura 1980) or (Taine 2003), it is advised to check that the code predicts a reasonable heat flux. Some simplified formulae are provided hereafter to evaluate the Nusselt number Nu (non-dimensional heat flux) in specific configurations for an incompressible flow, with ρ the density, T_∞ the temperature at infinity, λ the thermal conductivity, $Pr = \nu/a$ the Prandtl number (with $a=\lambda/(\rho C_p)$ and ν the kinematic viscosity), $\beta=-1/\rho\ (\partial\rho/\partial T)|_P$ the density variation with temperature at constant pressure and g the acceleration of gravity:

  - For the forced convection in a smooth pipe of length L and of hydraulic diameter D_h, with $L/D_h > 60$, for a turbulent flow with a Reynolds number $Re=UD_h/\nu$ such that 10 000 < Re < 120 000 and for a Prandtl number such that 0.7 < Pr < 100 (properties evaluated at $(T_\infty+T_p)/2$), the mean heat flux over the length L is $\Phi = Nu\ \lambda\ (T_\infty\text{-}T_p)/D_h$ with Nu computed as:

    $Nu = 0.023\ Re^{0.8}\ Pr^{1/3}$ (Colburn)

  - For the natural convection on a vertical flat plate of height L at a constant temperature T_p, with a Rayleigh number $Ra = g\ \beta\ \Delta T\ L^3/(\nu a)$ such that $10^9 < Ra < 10^{13}$ (properties evaluated at $(T_\infty+T_p)/2$), the mean heat flux is $\Phi = Nu\ \lambda\ (T_\infty\text{-}T_p)/L$ with Nu computed as:

    $Nu = 0.13\ Ra^{1/3}$ (Mac Adams)

- Jets: for an incompressible flow, a round jet emitted from an orifice into a large domain develops in two parts; first, a "potential" kernel cone (in which the velocity remains identical to the velocity at the orifice) progressively spreads out over a distance of approximately 8 times the diameter of the orifice; then, the velocity of the jet decreases as the inverse of the distance from the orifice (Viollet 1997 citing Abramovitch 1963).

  For the second part, where the centreline velocity decreases, correlations are available (Hug 1970), and may help to assess the relevance of the results (or to *a priori* determine the minimal size of the computational domain that is required so that the outlet is "sufficiently far away"). With z the distance from the orifice, d the diameter of the orifice, u_b the velocity of the jet at the orifice, c the concentration of a tracer such that $c=c_b=1$ in the jet at the orifice and $c=c_a=0$ the ambient concentration (for a constant $C_P=\partial h/\partial T|_P$, this concentration may represent a non-dimensional temperature T, that is $c=(T_a\text{-}T)/(T_a\text{-}T_b)$, with T_b the temperature in the jet at the orifice and T_a the ambient temperature), the maximal velocity u_m and the maximal concentration c_m on the centreline of the jet read:

$$u_m = 6.2u_b\ d/z$$

$$c_m = 5.6\ d/z$$

- Plane mixing layer: for an incompressible flow, the width e_{cm} of a mixing layer between two velocities $u_1 > u_2$ is proportional to the distance x (Viollet 1997 citing Papamoshkou and Roshko 1988):

$$e_{cm} = 0.17\ x\ (u_1-u_2)/(½(u_1+u_2))$$

- Backward facing step: for the flow behind a backward facing step with a Reynolds number based on the step height of 5100, the DNS of Le and Moin indicates that the reattachment point is located close to 6 times the height of the step (http://cfd.mace.manchester.ac.uk/ercoftac/).
- Vortex shedding behind a cylinder with a circular section[1] of diameter L: for a L-based Reynolds number ranging from 200 to 10 000, (Roshko 1953) measured a Strouhal number associated with the vortex shedding of approximately 0.21+/-0.01. For Reynolds numbers larger than 10 000, the Strouhal number increases slightly: (Roshko 1961) presented measurements of approximately 0.27 for a Reynolds number close to $5\ 10^6$.

References

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[1] For an infinite prismatic obstacle with a square section, the measured values are close to those observed for an infinite cylinder with a circular section. Hence, for Reynolds numbers ranging from 100 to 10 000, the Strouhal number *associated with the lift* is approximately 0.13 (Okajima 1982) and the Strouhal number associated with the drag – and hence with the vortex shedding – is close to 0.21, which is the value observed for an infinite cylinder with a circular section (the fact that the drag frequency is approximately twice as large as the lift frequency comes from the fact that for one period associated with the lift, two vortices must have detached from the obstacle: one from the upper part and one from the lower part; the drag, on the other hand, goes over one cycle each time a vortex is shed).