A two-dimensional code to simulate liquid film on the blades of steam turbines

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\textit{Code_Saturne} user meeting
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Outline

- Context and Objectives
- Liquid film model
- Simulations
- Conclusions and perspectives
Context and Objectives

Wetness in steam turbine

- Nucleation
- Growth
- Deposition
- Liquid film
- Atomization
Wetness induces **erosion** and **losses**

**Baumann’s rule (1912):**

\[ 1\% \text{ humidity} = 1\% \text{ power loss} \]

Describe the unsteady liquid film on stator and rotor blades with a model and numerical simulations
Liquid film model

Real phenomena

Model

\( \neq \) curved surface
Liquid film model

⇒ Literature analysis: thin (10 to 100 $\mu m$), laminar, continuous

1. Rivulets and dry zones
2. Continuous film with 2d waves at the free surface
3. Transition
4. Continuous film with 3d waves at the free surface
5. Continuous film with 3d waves at the free surface and atomisation

Hammitt et al., 1981, Forschung im Ingenieurwesen

⇒ Modified Shallow-Water equations: integral formulation of simplified Navier-Stokes equations with specific physics
Liquid film model

Frames

\[ \vec{\Omega} = \omega_0 \vec{x}_0 \]

(turbine shaft)

\[ \vec{g} = -g \vec{y}_0 \]

(Gas)

(Liquid)

\[ h(x, y, t) \]

\[ \theta \]

\[ z_b(x, y) \]

\[ n^- (x, y, t) \]
Liquid film model

Simplified incompressible N-S equations at order $O(\varepsilon^2)$ with $\varepsilon = h/l$.

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]

\[ \frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_{xz-r}}{\partial z} \]

\[ \frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = g_y - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\rho} \frac{\partial \tau_{yz-r}}{\partial z} + \omega_0^2 (r + y) + 2\omega_0 w \]

\[ \frac{\partial p}{\partial z} = \rho g_z - 2\rho \omega_0 v + \rho \omega_0^2 z \]

with $\tau_{xz-r} = \mu \frac{\partial u}{\partial z}$ and $\tau_{yz-r} = \mu \frac{\partial v}{\partial z}$
Liquid film model

**Boundary conditions**

- **At the wall:**
  \[ u_{|z=z_b} = 0; \ v_{|z=z_b} = 0; \ w_{|z=z_b} = 0 \]

- **At the free surface:**
  
  mass balance \Rightarrow \frac{dm}{dt} = \rho S_m dx dy
  
  \Rightarrow \frac{d(\eta - z)}{dt} = S_m
  
  \Rightarrow w_{|z=\eta} = \partial_t \eta + u_{|z=\eta} \partial_x \eta + v_{|z=\eta} \partial_y \eta - S_m
Liquid film model

Exact form of the integration over the thickness of the film

\[
\frac{\partial h}{\partial t} + \frac{\partial h\bar{u}}{\partial x} + \frac{\partial h\bar{v}}{\partial y} = S_m
\]

\[
\frac{\partial h\bar{u}}{\partial t} + \frac{\partial}{\partial x} \left( \int_{z_b}^{\eta} u^2 \, dz + \frac{g\cos(\theta) h^2}{2} \right) + \frac{\partial}{\partial y} \int_{z_b}^{\eta} u v \, dz + 2\omega_0 \int_{z_b}^{\eta} \frac{\partial}{\partial x} \int_{z_b}^{\eta} v \, dz \, dz - \frac{\omega_0^2}{2} h \frac{\partial \eta^2}{\partial x} =
\]

\[-h \frac{\partial \bar{P}_{\text{liquid}}}{\partial x} \bigg|_{z=\eta} - h g \cos(\theta) \frac{\partial z_b}{\partial x} + \frac{1}{\rho} \left( \tau_{xz-r} \bigg|_{\eta} - \tau_{xz-r} \bigg|_{z_b} \right) + u \bigg|_{\eta} S_m \]

\[
\frac{\partial h\bar{v}}{\partial t} + \frac{\partial}{\partial x} \int_{z_b}^{\eta} u v \, dz + \frac{\partial}{\partial y} \left( \int_{z_b}^{\eta} v^2 \, dz + \frac{g\cos(\theta) h^2}{2} \right) + 2\omega_0 \int_{z_b}^{\eta} \frac{\partial}{\partial y} \int_{z_b}^{\eta} v \, dz \, dz - \frac{\omega_0^2}{2} h \frac{\partial \eta^2}{\partial y} =
\]

\[g y h - h \frac{\partial \bar{P}_{\text{liquid}}}{\partial y} \bigg|_{z=\eta} - h g \cos(\theta) \frac{\partial z_b}{\partial y} + \frac{1}{\rho} \left( \tau_{yz-r} \bigg|_{\eta} - \tau_{yz-r} \bigg|_{z_b} \right) + 2\omega_0 h\bar{w} + h\omega_0^2 (r + y) + v \bigg|_{\eta} S_m \]

\[\Rightarrow 8 \text{ required closure laws (colored terms)} \]
**Liquid film model**

Closure laws: \[ \int_{z_b}^{\eta^-} u^2 dz, \int_{z_b}^{\eta^-} u v dz, \int_{z_b}^{\eta^-} v^2 dz \]

\[ \int_{z_b}^{\eta^-} u^2 dz = \Gamma h \bar{u}^2 \quad \text{with} \quad \bar{u} = \frac{1}{h} \int_{z_b}^{\eta^-} u dz \]

Legitimacy for parabolic velocity profile:
- **Theory**: Poiseuille flow
- **Experiments**: Falling films

For the model, \( \Gamma = 1 \) or \( \Gamma = 6/5 \)

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1. Bertshy et al., 1983, *JFM*
3. Adomeit, 2000, *IJMF*
Liquid film model

Closure laws: \( u_{\eta^-}, v_{\eta^-} \)

\[ \Rightarrow \text{Entropy equation for a global system composed with liquid and vapor} \]

Stating that:

- the velocity at the free surface depends on both phases
- the entropy of the system must increase in time
- the transfer of mass between the two phases is only due to thermodynamic phenomena
- the velocity of the liquid is represented by its mean velocity

the only entropy-consistent velocity is:

\[ u_{\eta^-} = \frac{\bar{u} + u_{\text{vapor}}}{2} \]

\[ \Rightarrow \text{Physical behavior} \]

\[ \begin{array}{cccccc}
\overset{u_{\text{gas}}}{\text{Gas}} & \quad & \overset{\text{Gas}}{?} & \quad & \overset{u_{\text{liquid}}}{\text{Liquid}} & \quad & \overset{z_b(x,y)}{\text{Liquid}} \\
\end{array} \]

\[ \eta^-(x,y,t) \]

13/24
Liquid film model

General model

\[
\begin{align*}
\frac{\partial h}{\partial t} + \frac{\partial h\bar{u}}{\partial x} + \frac{\partial h\bar{v}}{\partial y} &= S_m \\
\frac{\partial h\bar{u}}{\partial t} + \frac{\partial \left( \Gamma h\bar{u}^2 + \frac{g\cos(\theta)h^2}{2} \right)}{\partial x} + \frac{\partial \Gamma h\bar{u}}{\partial y} + 2\omega_0 \left( \bar{v}h \frac{\partial \eta}{\partial x} + \frac{h^2}{2} \frac{\partial \bar{v}}{\partial x} \right) - \frac{\omega_0^2}{2} h \frac{\partial \eta^2}{\partial x} = \\
&- \frac{h}{\rho} \frac{\partial P_{film}^{z=\eta}}{\partial x} - h\cos(\theta) \frac{\partial z_b}{\partial x} + \frac{1}{\rho} \left( \tau_{xz} - r|_\eta - \tau_{xz} - r|_{z_b} \right) + \left( \frac{\bar{u} + u_{gas}}{2} \right) S_m \\
\frac{\partial h\bar{v}}{\partial t} + \frac{\partial \Gamma h\bar{u}}{\partial x} + \frac{\partial \left( \Gamma h\bar{v}^2 + \frac{g\cos(\theta)h^2}{2} \right)}{\partial y} + 2\omega_0 \left( \bar{v}h \frac{\partial \eta}{\partial y} + \frac{h^2}{2} \frac{\partial \bar{v}}{\partial y} \right) - \frac{\omega_0^2}{2} h \frac{\partial \eta^2}{\partial y} + 2\omega_0 \frac{h^2}{2} \left( \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \right) = g\sin \theta h - \\
&\frac{h}{\rho} \frac{\partial P_{film}^{z=\eta}}{\partial y} - h\cos(\theta) \frac{\partial z_b}{\partial y} + \frac{1}{\rho} \left( \tau_{yz} - r|_\eta - \tau_{yz} - r|_{z_b} \right) + h\omega_0^2 (r + y) + 2\omega_0 h \left( \bar{u} \frac{\partial z_b}{\partial x} + \bar{v} \frac{\partial z_b}{\partial y} \right) + \left( \frac{\bar{v} + v_{gas}}{2} \right) S_m
\end{align*}
\]

<table>
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<tr>
<th></th>
<th>Stator model</th>
<th>Rotor model with $\Gamma = 1$</th>
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</thead>
<tbody>
<tr>
<td><strong>Hyperbolicity</strong></td>
<td>$\cos \theta &gt; 0$</td>
<td>$g\cos \theta + 2\omega_0 \bar{v} - \omega_0^2 z_b &gt; 0$</td>
</tr>
<tr>
<td><strong>Entropy-entropy flux couple</strong></td>
<td>$\Gamma = 1$</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Convex entropy</strong></td>
<td>$\cos \theta &gt; 0$</td>
<td>$g\cos \theta - \omega_0^2 \eta &gt; 0$</td>
</tr>
<tr>
<td><strong>Galilean invariance</strong></td>
<td>$\Gamma = 1$</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Rotational invariance</strong></td>
<td>yes</td>
<td>no (Coriolis)</td>
</tr>
</tbody>
</table>
Simulations - verification

Dam break (particular Riemann problem)

\[(h)_t + (h\bar{u})_x = 0\]

\[(h\bar{u})_t + \left( h\bar{u}^2 + \frac{g\cos(\theta)h^2}{2} \right)_x = 0\]

\[(h\bar{v})_t + (h\bar{u}\bar{v})_x = 0\]
Simulations - verification

Dam break with $\Gamma = 6/5$

\[(h)_t + (h \bar{u})_x = 0\]

\[(h \bar{u})_t + \left( \Gamma h \bar{u}^2 + \frac{g \cos(\theta) h^2}{2} \right)_x = 0\]

\[(h \bar{v})_t + (\Gamma h \bar{u} \bar{v})_x = 0\]

\[\lambda_1 = \Gamma U_n - c \sqrt{1 + \Gamma (\Gamma - 1) M^2}\]

\[\lambda_2 = \Gamma U_n\]

\[\lambda_3 = \Gamma U_n + c \sqrt{1 + \Gamma (\Gamma - 1) M^2}\]

\[\lambda_1 = \lambda_2 + \lambda_3\]

\[\lambda_1 = \lambda_2 + \lambda_3\]

\[\Rightarrow \text{No Riemann invariant found, so no analytical solution found.}\]
Simulations - verification

Hump (particular Riemann problem)

\[
(h)_t + (h u)_x = 0
\]

\[
(h u)_t + \left( h u^2 + \frac{g \cos(\theta) h^2}{2} \right)_x = 0
\]

Leveque, 2004
Simulations - verification

Inclined lake at rest

\[(h)_t + (h\bar{v})_y = 0\]

\[(h\bar{v})_t + \left(h\bar{v}^2 + \frac{g\cos(\theta)h^2}{2}\right)_y = ghsin(\theta)\]

The inclined lake at rest is an analytical solution of the model \((\bar{v} = 0 \text{ and } h = tan\theta + h_0)\). The code verify this solution as the initial condition do not evolve.
Simulations - validation

Falling liquid film on an inclined plate

- A small perturbation is amplified (unstable) if \( \text{Re}_{\text{film}} > \text{Re}_c \).
  - Theory: Linear stability analysis of N-S equations
  - Experiment: Liu and Gollub, 1994, PoF
  \[ \Rightarrow \text{Re}_c = \frac{5}{6} \cot(\theta) \]

- Model
  \[
  \begin{align*}
  (h)_t + (h\bar{v})_y &= 0 \\
  (h\bar{v})_t + \left( h\bar{v}^2 + \frac{g\cos(\theta)h^2}{2} \right)_y &= ghsin(\theta) - \frac{3\nu\bar{v}}{h}
  \end{align*}
  \]

- Relative height at the center versus time for \( \theta = 6.4 \Rightarrow \text{Re}_c = 7.4 \)

\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
<table>
<thead>
<tr>
<th>Cells number/Re</th>
<th>7.7</th>
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<th>7.9</th>
<th>8.0</th>
<th>8.1</th>
<th>8.2</th>
<th>8.3</th>
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<td>S</td>
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<td>U</td>
<td>U</td>
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<td>U</td>
</tr>
</tbody>
</table>
\end{tabular}

S for stable
U for unstable
Simulations - validation

Highly sheared film on a plate under steam turbine conditions

- **Experiment**\(^4\): 0.2 bar, 52.2 °C, steam velocity 60-390 m/s, film flow rate \(5.10^{-7} \text{ m}^3/\text{s}\)
  \[\Rightarrow\] Measured film height from 50 to 200 \(\mu\text{m}\)

- **Model**

\[
(h)_{t} + (h\vec{v})_{y} = S_{m}
\]

\[
(h\vec{v})_{t} + \left(h\vec{v}^2 + \frac{g\cos(\theta)h^2}{2}\right)_{y} = g\sin(\theta)h + \frac{1}{\rho} \left(\tau_{yz} - r_{\eta} - \tau_{yz} - r_{z_b}\right)
\]

---

\(^4\) Hammitt et al., 1975, *Report UMICH* and 1981, *Forschung im Ingenieurwesen*
Simulations

Highly sheared film on a plate under steam turbine conditions

with wall shear

(a) \[ \frac{3\mu}{h} \frac{\tau_{yz} - r}{2\eta} \]

(b) \[ \frac{c_f \rho v^2}{2} \text{ with } c_f = \frac{16}{Re} \]

(c) \[ \frac{c_f \rho v^2}{2} \text{ with } c_f = \frac{24}{Re} \]

with free surface shear

\[ c_{f \text{ int}} \rho |u_g - \bar{u}|(u_g - \bar{u}) \]

(1) \[ c_{f \text{ int}} = 0.005 \]

(2) \[ c_{f \text{ int}} = 0.0142 \]

(3) \[ c_{f \text{ int}} = 0.008 + 2 \times 10^{-5} Re \]

(4) \[ c_{f \text{ int}} = \left(0.0007 + 0.0625Re^{-0.32}\right)(1 + 0.025Re) \]

Wall shear (a)

Wall shear (b)

Wall shear (c)
Simulations

Implementation of the surface tension ($\frac{\sigma}{\rho} h \partial_{yyy} h$)

Option 1:
- unchanged model
- centered scheme

Option 2 (Noble & Vila, 2013):
- extended model (+ 1 equation which reduced the order of the system)
- centered scheme

Model (option 1):

$$(h)_t + (h \vec{v})_y = 0$$

$$(h \vec{v})_t + \left( h \vec{v}^2 + \frac{g \cos(\theta) h^2}{2} \right)_y = g h \sin(\theta) - \frac{3 \nu \vec{v}}{h} + \frac{\sigma}{\rho} h \partial_{yyy} h$$

Falling film: linear stability analysis
$Re_{film} = 8.26$, $\theta = 6.4^\circ$ and 100 nodes
A liquid film flowing down the bottom side of an inclined plate is always unstable. This instability can be absolute or convective depending on the Kapitza number (surface tension/inertia), the Reynolds number (viscosity/inertia) and the angle $\theta$. Kofman, PoF, 2016

\[
(h)_{,t} + (h\nu)_{,y} = 0
\]

\[
(h\nu)_{,t} + \left( h\nu^2 + \frac{g\cos(\theta)h^2}{2} \right)_{,y} = g\sin(\theta) - \frac{3\nu\nu}{h} + \frac{\sigma}{\rho} h\partial_{yyy} h
\]
Conclusions and perspectives

• A model (stator and rotor) for liquid film in steam turbines has been proposed and its properties have been examined.

• The entire stator model has been implemented (convection, mass transfer, shear at the wall and at the free surface, surface tension, gas pressure, droplet impact momentum)

• The stator model has been verified with two Riemann problems, the inclined lake at rest and validated with the falling film, the highly sheared film and the inverted gravity experiments.

• Implementation of rotational effect.

• Chaining method (Fendler and Blondel’s results)

Thank you for your attention.